Indian J. Phys. **72B** (1), 45-54 (1998)

IJP B
— an international journal

The hydromagnetic convective flow through a vertical channel

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Received 23 May 1997, accepted 24 September 1997

Abstract : The radiation and Hall effects on the combined free and forced convection flow of an electrical conducting viscous fluid through an open-ended vertical channel permeated by a uniform transverse magnetic filed, in slip flow regime, have been considered. The temperature on the walls has been supposed to vary linearly with distance On the assumption of optically thin limit, the expressions for velocity, induced magnetic field and temperature are obtained and the influence of Hall currents and radiation on these solutions are graphically shown.

Keywords : Flow in channels, radiative heat transfer, Hall currents

PACS Nos. 47.60+i, 44.40.+a, 47.27.Te

1. Introduction

The problems concerning the heat transfer in electrically conducting fluid permeated by electromagnetic field have been studied by many authors due to its applications in the magnetohydrodynamic design generators, cross field accelerators, pumps *etc*. The first step was made by Siegel [1] and Roming [2] who considered the forced convection of an electrically conducting fluid flowing in a channel in the presence of a transverse magnetic field. Grief *et al* [3] obtained the exact solutions for the problem of fully developed laminar convection flow of a radiating gas in a vertical channel in optically thin limit.

Because of its great importnace in space applications and higher operating temperature, the problem of radiation on heat transfer was developed by Gupta and Gupta [4] and Sanyal and Samanta [5]. They studied the radiation effects in hydromagnetic convection in a vertical open-ended channel permeated by a constant magnetic field in transverse direction.

The purpose of this paper is to take into consideration the Hall and radiation effects on the combined free and forced convection flow of an electrically conducting rarefied gas through an open ended vertical channel in the presence of a uniform magnetic field.

Assuming the slip flow regime and confining the analysis to the optically thin limit, the velocity, induced magnetic field and temperature solutions are obtained and the influence of Hall currents and radiation on these solutions are graphically shown.

2. Fundamental equations

Let us consider a fully-developed steady state hydromagnetic laminar flow of an electrically conducting incompressible and viscous fluid through an open-ended vertical channel. This channel is permeated by a uniform magnetic field B_0, in the direction normal to the plates at the distance 21 apart. The plates of this channel are non-conducting and the surface temperature of the plates is assumed to vary along the vertical direction. Let the center of the channel be taken as the origin, the z-axis along the vertical direction and x-axis along the direction normal to the plates. For fully developed laminar flow in a uniform magnetic field, in the presence of radiation and Hall effects, all physical quantities except temperature and pressure are functions of x-alone.

The momentum, energy Maxwell and modified Ohm's law (including the Hall effect) are

$$\frac{\partial v}{\partial t}+(v\nabla).v=-\frac{1}{\rho}\nabla p+\frac{1}{\mu_0\rho}(\nabla\times B)\times B+\nu\nabla v-\alpha g\theta\lambda, \tag{1}$$

$$\frac{\partial\theta}{\partial t}+(v\nabla).\theta=k\nabla\theta-\frac{1}{pC_p}\nabla\Phi_R, \tag{2}$$

$$j=\sigma[E+v\times B]-\frac{\omega\tau}{B_n}j\times B, \tag{3}$$

where $v(0,u,w)$ is the velocity, $B(B_0, b_x, b_z)$ the magnetic induction, ρ the fluid density, p the pressure, μ_0 the magnetic permeability, ν the kinematics viscosity, α the coefficient of volume expansion g the acceleration due to gravity, $\lambda(0,0,-1)$ the unit vector of vertical direction, k the thermal diffusivity, C_p the specific heat at the constant pressure, σ the electrical conductivity, $\theta = T - T_0$, T being the temperature and T_0 the reference temperature (taken as constant), Φ_R the radiative heat flux, E the electric intensity, ω the electron Larmor frequency and τ the mean free interval between the successive collisions of an electron with ions.

In this model, we shall neglect the viscous and ohmic dissipation and assuming that the temperature inside the fluid has the vertical gradient β (*a* const.), we can take [4]

$$T=T^*(x)+\beta.z. \tag{4}$$

If we take

$$\theta^*=T^*(x)-T_0 \tag{5}$$

from (5) we can find

$$\theta=\theta^*(x)+\beta z. \tag{6}$$

With these assumptions, eq. (2) can be written in the form

$$w\beta = k\frac{d^2\theta^*}{dx^2} - \frac{1}{pC_p}\frac{\partial\Phi_R}{\partial x}. \tag{7}$$

For an optically thin limit and for a non-gray gas near equilibrium, using the relation

$$A = \frac{4}{pC_p}\int_0^\infty \chi\lambda_0\left(\frac{de_{b_2}}{dT}\right)_0 d\lambda, \tag{8}$$

where $\chi\lambda_0$ is the absorption coefficient, e_{b_2} the Planck function and the subscript 0(zero) indicates that the quantities have been evaluated at the reference temperature T_0, it is easy to see that the eq. (8) gives

$$\beta w = k\frac{d^2\theta^*}{dx^2} - A\theta^*;\ A\theta^* = \frac{1}{pC_p}\frac{\partial\phi_R}{\partial x} \tag{9}$$

The momentum and magnetic induction equations together with (7) are simply reduced to

$$\frac{\partial p}{\partial x} + \frac{1}{2\mu_0}\frac{d}{dx}\left(b_y^2 + b_z^2\right) = 0, \tag{10}$$

$$\nu\frac{d^2u}{dx^2} + \frac{B_0}{\mu_0\rho}\frac{db_y}{dx} - \frac{1}{\rho}\frac{\partial p}{\partial y} = 0, \tag{11}$$

$$\nu\frac{d^2w}{dx^2} + \frac{B_0}{\mu_0\rho}\frac{db_z}{dx} + g\alpha(\theta^* + \beta.z) - \frac{1}{\rho}\frac{\partial p}{\partial z} = 0, \tag{12}$$

$$v_m\left(\frac{d^2b_y}{dx^2} + \omega\tau\frac{d^2b_z}{dx^2}\right) + B_0\frac{du}{dx} = 0, \tag{13}$$

$$v_m\left(\frac{d^2b_z}{dx^2} - \omega\tau\frac{d^2b_y}{dx^2}\right) + B_0\frac{dw}{dx} = 0, \tag{14}$$

where $v_m = \dfrac{1}{\sigma\mu_0}$ is the magnetic viscosity.

If $p = p(x, z)$ is not dependent on the depth of the channel and $\dfrac{\partial p}{\partial y} = 0$, we can integrate eq. (10) with respect to x and get

$$p = -\frac{1}{2\mu_0}(b_y^2 + b_z^2) + f(z). \tag{15}$$

If we substitute this into eq. (12), we have

$$\nu\frac{d^2w}{dx^2}+\frac{B_0}{\mu_0\rho}\frac{db_z}{dx}+g\alpha\theta^* = \frac{1}{\rho}\frac{\partial p}{\partial z}-g\alpha\beta z = C,$$

where C is a constant depending on the physics of the problem. It may be determined from the end conditions of pressure to which the channel is subjected (or from the mass flow through the channel).

Thus eqs. (11) and (12) becomes

$$\nu\frac{d^2u}{dx^2}+\frac{B_0}{\mu_0\rho}\frac{db_y}{dx}=0, \tag{16}$$

$$\nu\frac{d^2w}{dx^2}+\frac{B_0}{\mu_0\rho}\frac{db_z}{dx}+g\alpha\theta^*=C, \tag{17}$$

Introducing the non-dimensional quantities

$$x^*=\frac{x}{l};u^*=\frac{ul}{k};w^*\frac{lw}{k};t=-\frac{\theta^*}{\beta l};b_y^*=\frac{b_y}{B_0}; \tag{18}$$

$$b_z^*=\frac{b_z}{B_0};H=B_0l\sqrt{\frac{1}{\mu_0\rho\nu k}};R_0\frac{\alpha g\beta l^4}{\nu k};P=\mu_0\sigma k;$$

the eqs. (16), (17), (14), (13) and (9) reduce to

$$\frac{d^2u^*}{dx^{*2}}+H^2\frac{db_y^*}{dx^*}=0, \tag{19}$$

$$\frac{d^2w^*}{dx^{*2}}+H^2\frac{db_z^*}{dx^*}-R_at=C_1;C_1=\frac{Cl^3}{\nu k}, \tag{20}$$

$$\frac{d^2b_y^*}{dx^{*2}}+\omega\tau\frac{d^2b_z^*}{dx^{*2}}=-P\frac{du^*}{dx^*}, \tag{21}$$

$$\omega\tau\frac{d^2b_y^*}{dx^{*2}}-\frac{d^2b_z^*}{dx^{*2}}=P\frac{dw^*}{dx^*}, \tag{22}$$

$$w^*=-\frac{d^2t}{dx^{*2}}+F.t;F=\frac{Al^2}{k}. \tag{23}$$

Elimination of u^*, w^*, b_y^* and b_z^* from eqs. (19–23) yields

$$\frac{d^7 t}{dx^{*7}} - k_1 \frac{d^5 t}{dx^{*5}} + k_2 \frac{d^3 t}{dx^{*3}} - k_3 \frac{d t}{dx^*} = 0, \tag{24}$$

where

$$k_1 = F + \frac{2H^2 P}{1+\omega^2\tau^2},$$

$$k_2 = R_a + \frac{H^2 P}{1+\omega^2\tau^2}(2F + H^2 P),$$

$$k_3 = \frac{H^2 P}{1+\omega^2\tau^2}(R_a + H^2 PF).$$

The solution for temperature $t(x^*)$ permit to integrate eqs. (23) and (20) to obtain the solutions for $w^*(x^*)$ and $b_z^*(x^*)$. The other solutions can be obtained by integration of eqs. (22) and (19).

3. Solution of the problem

Since the walls of the channel are non conducting, the boundary condition is [6]

$$b = 0 \text{ at } x^* = \pm 1. \tag{25}$$

The first order velocity slip and temperature jump conditions, neglecting the thermal creep term are [7]

$$v = \mp h \frac{dv}{dx^*} \text{ at } x^* = \pm 1, \tag{26}$$

$$t = \mp f \frac{dt}{dx^*} \text{ at } x^* = \pm 1, \tag{27}$$

where $h = \frac{(2-f_1)}{f_1} . L_1$, $f = \frac{2-f}{f_2} . \frac{2\xi}{\xi+1} . \frac{L_1}{P}$, f_1 being the reflection coefficient, L_1 is the mean free path (is a constant for incompressible fluid) [7], f_2 is the thermal accommodation coefficient, ξ is the specific heat ratio and P is the Prandtl number. In this case we shall take h and f as constants.

The solutions for temperature $t(x^*)$, velocities $w^*(x^*)$, $u^*(x^*)$ and induced magnetic fields $b_z^*(x^*)$, $b_y^*(x^*)$ have the form

$$t(x^*) = 1 + 2. D.\cosh(kx^*) + 2. M.\cos(\beta_1 x^*).\cosh(\alpha_1 x^*) + \\ + 2. N.\sin(\beta_1 x^*).\sinh(\alpha_1 x^*) \tag{28}$$

$$
\begin{aligned}
w^*(x^*) = {} & F + 2.(F - k^2).D.\cosh(kx^*) + M.[2(F - \alpha_1{}^2 + \beta_1{}^2).\cos(\beta_1 x^*). \\
& \cosh(\alpha_1 x^*) + 4\alpha_1\beta_1.\sin(\beta_1 x^*).\sinh(\alpha_1 x^*)] + \mathrm{N}.[2(\mathrm{F} - \alpha_1{}^2 + \beta_1{}^2). \\
& \sin(\beta_1 x^*).\sinh(\alpha_1 x^*) - 4\alpha_1\beta_1.\cos(\beta_1 x^*).\cosh(\alpha_1 x^*)],
\end{aligned} \tag{29}
$$

$$
\begin{aligned}
u^*(x^*) = {} & \frac{2}{H^2}\{2.d.(Fk^2 - k^4 - R_\alpha).\cos(kx^*) - H^2 D.(F + k^2).\cosh(kx^*) + \\
& +[-2M(\alpha_1^4 + \beta_1^4) + (2FM + H^2 M - 8N\alpha_1\beta_1)(\alpha_1^2 - \beta_1^2) + \\
& +2(6\alpha_1\beta_1 M + N(2F + H^2))\alpha_1\beta_1 - M(2R_\alpha + FH^2)].\cos(\beta_1 x^*)\cosh(\alpha_1 x^*) + \\
& +[-2N(\alpha_1^4 + \beta_1^4) + (2FM + H^2 M + 8M\alpha_1\beta_1)(\alpha_1^2 - \beta_1^2) + \\
& +2(6\alpha_1\beta_1 N - M(2F + H^2))\alpha_1\beta_1 - N(2R_\alpha + FH^2)].\sin(\beta_1 x^*)\sinh(\alpha_1 x^*)\},
\end{aligned} \tag{30}
$$

$$
\begin{aligned}
b_2{}^*(x^*) = {} & \frac{1}{H^2}\{(R_\alpha + 1)x^* + 2\left[\frac{R_\alpha}{k} - (F - k^2)k\right].D.\sinh(kx^*) + \\
& +2M\left(\frac{\alpha_1 R_\alpha}{\alpha_1{}^2 + \beta_1{}^2} - \alpha_1 F + \alpha_1^3 - 3\alpha_1\beta_1^2\right).\cos(\beta_1 x^*).\sinh(\alpha_1 x^*) \\
& \left[\left(\frac{\beta_1 R_\alpha}{\alpha_1^2 + \beta_1^2} - \beta_1 F + \beta_1^3 - 3\alpha_1^2\beta_1\right).\sin(\beta_1 x^*).\cosh(\alpha_1 x^*)\right. \\
& +2N\left[\left(\frac{\alpha_1 R_\alpha}{\alpha_1^2 + \beta_1^2} - \alpha_1 F + \alpha_1^3 - 3\alpha_1\beta_1^2\right).\sin(\beta_1 x^*).\cosh(\alpha_1 x^*)\right. \\
& \left[\left(\frac{\beta_1 R_\alpha}{\alpha_1^2 + \beta_1^2} + \beta_1 F + \beta_1^3 - 3\alpha_1^2\beta_1\right).\cos(\beta_1 x^*).\sinh(\alpha_1 x^*)\right.
\end{aligned} \tag{31}
$$

$$
\begin{aligned}
b_y{}^*(x^*) = {} & \frac{2}{H^4}\{2Dk(Fk^2 - 2k^4 - R_a).\sin(kx^*) + H^2 D(F + k^3).\sinh(kx^*) + \\
& +[N\alpha_1(FH^2 + 2R_\alpha + \alpha_1^2(2\alpha_1^2 - 2F - H^2)) - M\beta_1(FH^2 + 2R_\alpha + \beta_1^2 + 2F + \\
& +H^2)) + \alpha_1\beta_1(10(\beta_1^3 N - \alpha_1^3 M) + 20\alpha_1\beta_1(\beta_1 M - \alpha_1 N) + 3(2F + H^2)(\alpha_1 M + \beta_1 N)]. \\
& .\sin(\beta_1 x^*).\cosh(\alpha_1 x^*) + [M\alpha_1(FH^2 + 2R_\alpha + \alpha_1^2 - 2F - H^2)) + N\beta_1(FH^2 + \\
& +2R_\alpha + \beta_1^2(2\beta_1^2 + 2F + H^2)) + \alpha_1\beta_1(10(\beta_1^3 M - \alpha_1^3 N) - 20\alpha_1\beta_1(\beta_1 N - \alpha_1 M) + \\
& -3(2F + H^2)(\alpha_1 N + \beta_1 M)].\cos(\beta_1 x^*).\sinh(\alpha_1 x^*)\},
\end{aligned} \tag{32}
$$

where values of different parameters are tabulated below.

F	H^2	α_1	β_1	k
1		1 89746	0 821488	1.07189
2	6	1 90671	0 825676	1 44650
3		1 91818	0 824287	1 73205
1		2 45277	1 03179	1 04738
2	10	2.45454	1 03357	1.44464
3		2.45714	1.03497	1 75135

F	H^2	D	M	N
1		– 0 6578	0 1381	0.0985
2	6	– 0 6499	0 1755	0 2262
3		– 0 5939	0 1458	0 366
1		– 0 5029	0 0437	0 0362
2	10	– 0 4671	0 065	0 0757
3		– 0.4552	0.0648	0.1768

4. Discussion

The effect of radiation is to increase the rate of energy transport to the fluid, thereby increasing the temperature of the fluid. For a given value of the Rayleigh number ($Ra = 1$), the increasing of radiation parameter F and Hartmann number H results in the flatter temperature profiles. Thus, the radiation and Hall effects strongly reduce the temperature difference between the fluid and the channel walls and even the influene of natural convection. By taking $h = 0.2$, $f = 0.5$, $\omega t = 1$, $F = 1,2,3$, $H^2 = 6,10$, $P = 1$ these effects have been shown in Figures 1 and 2.

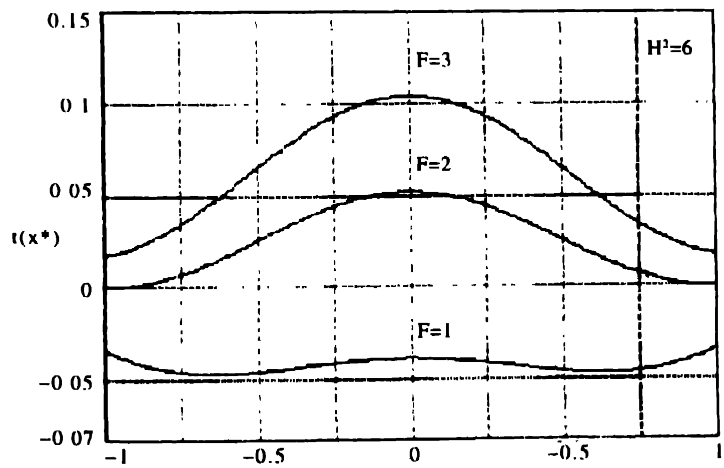

Figure 1. Variation of temperature $t(x^*)$ for different $x^*$and several values of radiation parameter F (1; 2; 3) when $H^2 = 6$.

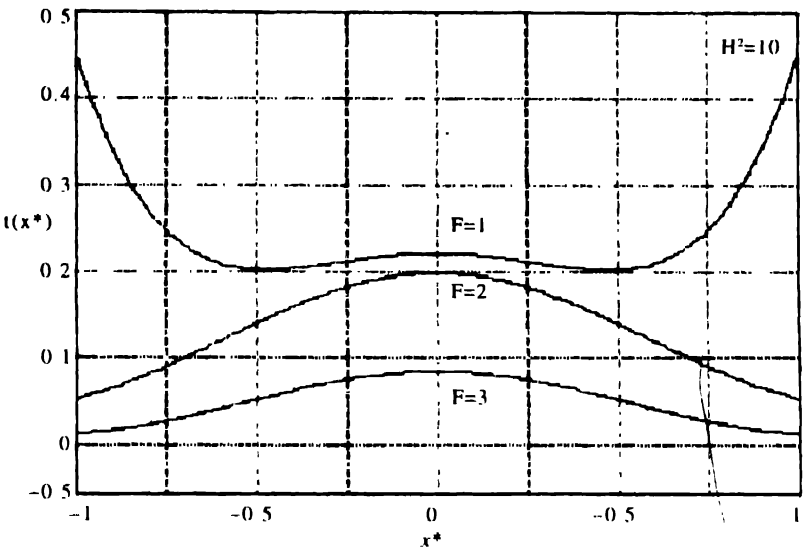

Figure 2. Variation of temperature $t(x^*)$ for different $x^*$and several values of radiation parameter F (1, 2, 3) when $H^2 = 10$

Due to these combined effects, for $Ra = 1$, there is a significant increase in the flow rate and so, the velocity $w^*(x^*)$ are greater than those resulting for the case of corresponding problem but without radiation and Hall effects [3]. This is shown in Figures 3 and 4.

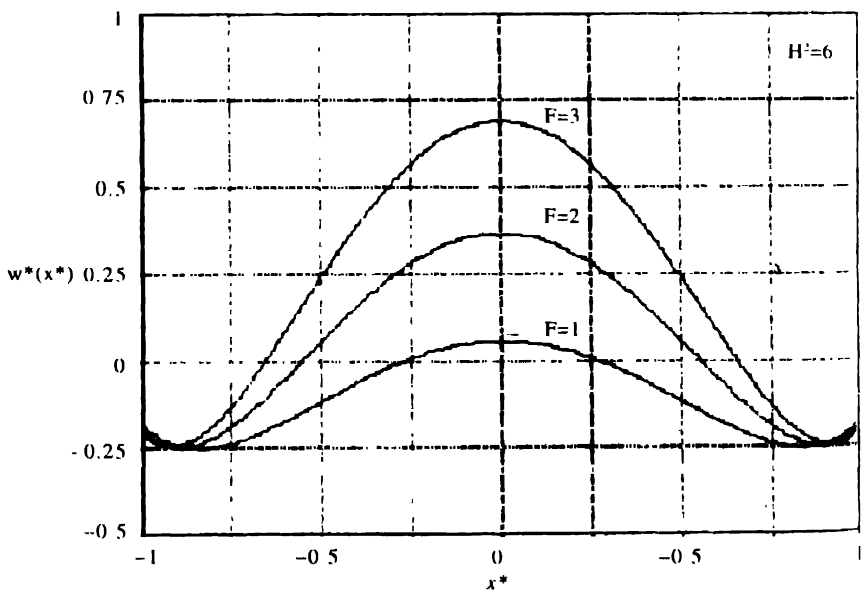

Figure 3. The dependence on x^* for various assigned values of F (1, 2; 3) of the velocity component $w^*(x^*)$ when $H^2 = 6$

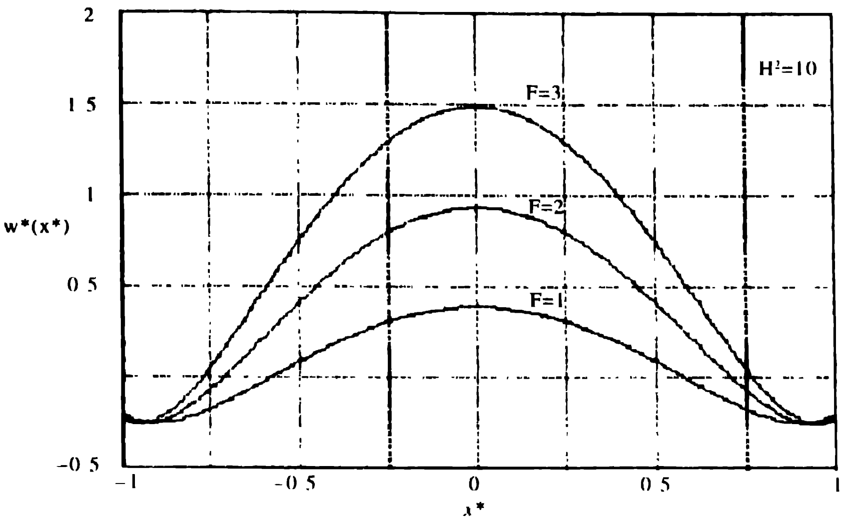

Figure 4. The dependence on x^* for various assigned values of F (1, 2, 3) of the velocity component $w^*(x^*)$ when $H^2 = 10$

In the case of induced magnetic field (Figures 5, 6), it is observed that $b_z^*(x^*)$ decreases with increasing F and H^2 down to $x^* = -1$ from $x^* = 0$ and then increases with F and is zero again at $x^* = 1$.

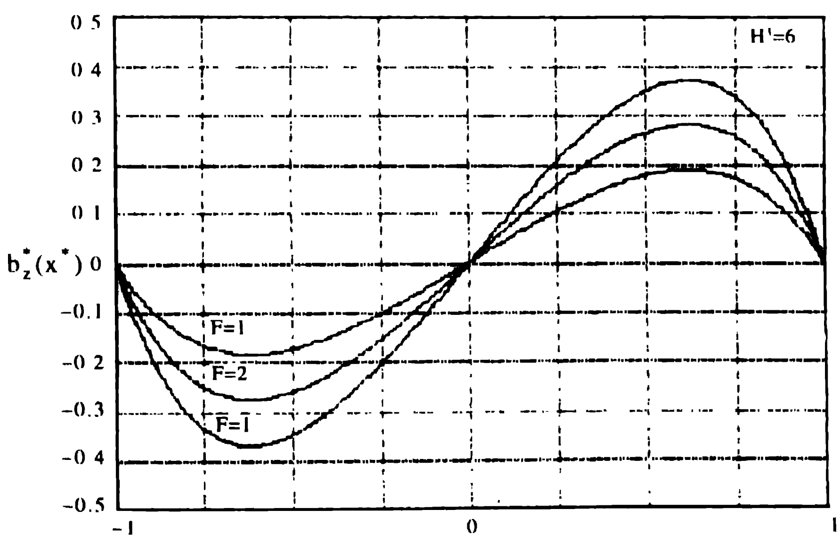

Figure 5. The magnetic field component b_z^* as a function of x^* for various assigned values of F (1; 2; 3) and $H^2 = 6$.

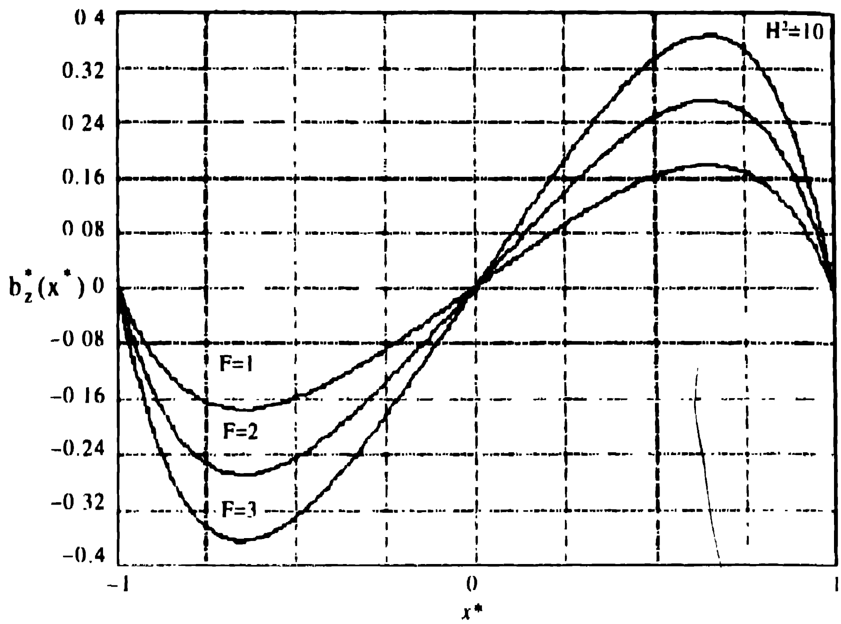

Figure 6. The magnetic field component b_z^* as a function of x^* for various assigned values of F (1, 2, 3) and $H^2 = 10$

The case of conducting walls will be the subject of another future paper.

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